Памятка. Теоремы о равносильных преобразованиях в уравнениях и неравенствах.

Repetitio est mater studiorum

Старинная латинская пословица.

1) $\frac{A(x)}{B(x)} = 0 \Leftrightarrow \begin{cases} A(x) = 0; \\ B(x) \neq 0 \end{cases}$

2) $A(x) \cdot B(x) = 0 \Leftrightarrow \begin{cases} \left[\begin{array}{l} A(x) = 0; \\ B(x) = 0 \end{array}\right. \\ \text{ОДЗ} \end{cases}$

3) $|A(x)| = B(x) \Leftrightarrow \begin{cases} \left[\begin{array}{l} A(x) = B(x); \\ A(x) = -B(x); \end{array}\right. \\ B(x) \geq 0 \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} A(x) \geq 0; \\ A(x) = B(x) \end{cases} \\ \begin{cases} A(x) < 0; \\ A(x) = -B(x) \end{cases} \end{array}\right.$

4) $|A(x)| = |B(x)| \Leftrightarrow \left(A(x)\right)^2 = \left(B(x)\right)^2 \Leftrightarrow \left(A(x) - B(x)\right)\left(A(x) + B(x)\right) = 0 \Leftrightarrow \left[\begin{array}{l} A(x) = B(x) \\ A(x) = -B(x) \end{array}\right.;$

5) $k_1|A_1(x)| + k_2|A_2(x)| + \cdots + k_s|A_s(x)| = B(x) \Leftrightarrow \left[\begin{array}{l} \begin{cases} x \leq x_1; \\ \ldots \end{cases} \\ \begin{cases} x_1 < x \leq x_2; \\ \ldots. \end{cases} \\ \ldots \\ \begin{cases} x \geq x_s; \\ \ldots. \end{cases} \end{array}\right.$

6) $|A(x)| >/\geq B(x) \Leftrightarrow \left[\begin{array}{l} A(x) >/\geq B(x) \\ A(x) </\leq -B(x) \end{array}\right.;$

7) $|A(x)| </\leq B(x) \Leftrightarrow \begin{cases} A(x) </\leq B(x); \\ A(x) >/\geq -B(x) \end{cases}$

8) $|A(x)| >/\geq/</\leq |B(x)| \Leftrightarrow \left(A(x)\right)^2 - \left(B(x)\right)^2 >/\geq/</\leq 0 \Leftrightarrow \left(A(x) - B(x)\right)\left(A(x) + B(x)\right) >/\geq/</\leq 0..$

9) $|A(x)|B(x) > 0 \Leftrightarrow \begin{cases} B(x) > 0; \\ A(x) \neq 0 \end{cases}$

10) $|A(x)|B(x) < 0 \Leftrightarrow \begin{cases} B(x) < 0; \\ A(x) \neq 0 \end{cases}$

11) $|A(x)|B(x) \leq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} B(x) \leq 0; \\ \text{ОГР}_{A(x)} \end{cases} \\ \begin{cases} A(x) = 0; \\ \text{ОГР}_{B(x)} \end{cases} \end{array}\right.$

12) $|A(x)|B(x) \geq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} B(x) \geq 0; \\ \text{ОГР}_{A(x)} \end{cases} \\ \begin{cases} A(x) = 0; \\ \text{ОГР}_{B(x)} \end{cases} \end{array}\right.$

13) $\frac{|A(x)|}{B(x)} > 0 \Leftrightarrow \begin{cases} B(x) > 0; \\ A(x) \neq 0 \end{cases}$

14) $\frac{|A(x)|}{B(x)} < 0 \Leftrightarrow \begin{cases} B(x) < 0; \\ A(x) \neq 0 \end{cases}$

15) $\frac{|A(x)|}{B(x)} \geq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} B(x) > 0; \\ \text{ОГР}_{A(x)} \end{cases} \\ \begin{cases} A(x) = 0; \\ B(x) \neq 0; \\ \text{ОГР}_{B(x)} \end{cases} \end{array}\right.$

16) $\frac{|A(x)|}{B(x)} \leq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} B(x) < 0; \\ \text{ОГР}_{A(x)} \end{cases} \\ \begin{cases} A(x) = 0; \\ B(x) \neq 0; \\ \text{ОГР}_{B(x)} \end{cases} \end{array}\right.$

17) $\sqrt[2k]{f(x)} = g(x) \Leftrightarrow \begin{cases} f(x) = \left(g(x)\right)^{2k}; \\ g(x) \geq 0 \end{cases}$

18) $\sqrt[2k]{f(x)} = \sqrt[2k]{g(x)} \Leftrightarrow \begin{cases} f(x) = g(x); \\ g(x) \geq 0 \end{cases}$

19) $\sqrt[2k]{f(x)} \cdot g(x) = 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} g(x) = 0; \\ f(x) \geq 0 \end{cases} \\ \begin{cases} f(x) = 0; \\ \text{ОГР}_{g(x)} \end{cases} \end{array}\right.$

20) $\frac{\sqrt[2k]{f(x)}}{g(x)} = 0 \Leftrightarrow \begin{cases} f(x) = 0; \\ g(x) \neq 0 \end{cases}$

21) $\sqrt[2k]{\mathrm{f(x)}} > g(x) \Leftrightarrow \left[\begin{array}{l} \begin{cases} f(x) > (g(x))^{2k}; \\ g(x) \geq 0 \end{cases} \\ \begin{cases} f(x) \geq 0; \\ g(x) < 0 \end{cases} \end{array}\right.$

22) $\sqrt[2k]{f(x)} \geq g(x) \Leftrightarrow \left[\begin{array}{l} \begin{cases} f(x) \geq (g(x))^{2k}; \\ g(x) \geq 0 \end{cases} \\ \begin{cases} f(x) \geq 0; \\ g(x) < 0 \end{cases} \end{array}\right.$

23) $\sqrt[2k]{f(x)} < g(x) \Leftrightarrow \begin{cases} f(x) < (g(x))^{2k}; \\ f(x) \geq 0; \\ g(x) > 0 \end{cases}$

24) $\sqrt[2k]{f(x)} \leq g(x) \Leftrightarrow \begin{cases} f(x) \leq (g(x))^{2k}; \\ f(x) \geq 0; \\ g(x) \geq 0 \end{cases}$

25) $\frac{\sqrt[2k]{f(x)}}{g(x)} > 0 / \sqrt[2k]{f(x)} \cdot g(x) > 0 \Leftrightarrow \begin{cases} f(x) > 0; \\ g(x) > 0 \end{cases}$

26) $\frac{\sqrt[2k]{f(x)}}{g(x)} < 0 / \sqrt[2k]{f(x)} \cdot g(x) < 0 \Leftrightarrow \begin{cases} f(x) > 0; \\ g(x) < 0 \end{cases}$

27) $\frac{\sqrt[2k]{f(x)}}{g(x)} \geq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} g(x) > 0; \\ f(x) \geq 0 \end{cases} \\ \begin{cases} f(x) = 0; \\ g(x) \neq 0; \\ \text{ОГР}_{g(x)} \end{cases} \end{array}\right.$

28) $\frac{\sqrt[2k]{f(x)}}{g(x)} \leq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} g(x) < 0; \\ f(x) \geq 0 \end{cases} \\ \begin{cases} f(x) = 0; \\ g(x) \neq 0; \\ \text{ОГР}_{g(x)} \end{cases} \end{array}\right.$

29) $\sqrt[2k+1]{f(x)} = g(x) \Leftrightarrow f(x) = (g(x))^{2k+1}$

30) $\sqrt[2k]{f(x)} \cdot g(x) \geq / \leq 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} f(x) \geq 0; \\ g(x) \geq / \leq 0 \end{cases} \\ \begin{cases} f(x) = 0; \\ \text{ОГР}_{g(x)} \end{cases} \end{array}\right.$